Precalculus Diagnostic Test

Mathematics Department

February 9, 2008

Instructions: Click the "Begin Assessment" button before you begin your selections. For each question, click the checkbox containing the "best" answer to the question. When you complete the assessment, click the "End Assessment" button with your mouse to obtain your results.

You can also obtain corrections to the assessment by clicking the "Correct" button. Answers are marked according to the following legend.

Legend: A ✔ indicates that the assessment-taker gave the correct response. A ✘ indicates an incorrect response. In this case, the correct answer is marked with a ●. You can examine solutions by clicking the correct answer marker ●.

Begin Assessment

1. If $x = -5$, then $|x - 3| - |4 - 3x| =$

☐ 10 ☐ -6 ☐ -8

☐ -11 ☐ -12

2. $(2x^2y^3)(-3x^3y^2)^3$ equals

☐ $-18x^{11}y^9$ ☐ $-18x^8y^8$ ☐ $-54x^8y^8$

☐ $-216x^{15}y^{15}$ ☐ $-54x^{11}y^9$

3. Jane can paint a room by herself in 5 hours. Working together with her sister Liz, they can paint the room in 3 hours. Which of the following equations best models the problem statement, where L represents the time it takes Liz to paing the room by herself?

☐ $5+3=8$ ☐ $\frac{3}{5}+\frac{3}{L}=1$ ☐ $\frac{1}{5}+\frac{1}{L}=5$

☐ $3+L=5$ ☐ $\frac{3}{5}+\frac{3}{L}=5$

4. If $f(x)=2x+3$ and $g(x)=3-2x$, then $f(g(3))$ equals

☐ -3 ☐ -1 ☐ 0

☐ 2 ☐ 4

5. If $a=2$ and $b=-7$, then

$$\frac{a^2-2ab+b^2}{a^2-b^2}$$

equals

☐ $-5/9$ ☐ $-9/5$ ☐ $-14/3$

☐ $-22/9$ ☐ $-11/3$

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6. If

$$x - y = a$$
$$x + 2y = b,$$

then y equals

- ☐ $\frac{a-b}{3}$
- ☐ $\frac{1+b}{3}$
- ☐ $\frac{a-3b}{3}$
- ☐ $\frac{b-a}{3}$
- ☐ $\frac{b-2a}{3}$

7. If

$$\sqrt[3]{3x-1} = -a,$$

then x equals

- ☐ a^3+1
- ☐ $\frac{a^3+1}{3}$
- ☐ $\frac{1-a^3}{3}$
- ☐ $3a^3+1$
- ☐ $1-3a^3$

8. A bag contins 90 marbles, each of which is either blue or red in color. If the ratio of the number of red marbles to blue marbles is 5 to 1, how many blue marbles are there?

- ☐ 70
- ☐ 15
- ☐ 20
- ☐ 12
- ☐ 18

9. One of the roots of

$$6x^2 - x - 15 = 0$$

is

- ☐ $3/2$
- ☐ $-3/8$
- ☐ $-3/5$
- ☐ $5/3$
- ☐ $4/5$

10. Simplify

$$\frac{a^2 + 2ab + b^2}{a^2 - b^2} \cdot \frac{a^2 - ab}{5a + 5b}.$$

- ☐ $\dfrac{a+b}{5}$
- ☐ $\dfrac{a-b}{5}$
- ☐ $\dfrac{a-b}{5a+5b}$
- ☐ $\dfrac{a}{5a+5b}$
- ☐ $\dfrac{a}{5}$

11. The inequality $-x > 4$ is equivalent to

- ☐ $x > -4$
- ☐ $x > 5$
- ☐ $x < -4$
- ☐ $x < 5$
- ☐ $x > -1/4$

12. $(4x^6y^{-4})^{1/2}$ equals

- ☐ $-\dfrac{2x^2}{y^3}$
- ☐ $\dfrac{2x^3}{y^2}$
- ☐ $-\dfrac{2x^3}{y^2}$
- ☐ $\dfrac{2x^2}{y^3}$
- ☐ $\dfrac{x^3}{2y^2}$

13. For $x < 0$, $\sqrt{25x^3}$ equals

☐ $5x\sqrt{x}$ ☐ $x\sqrt{5x}$ ☐ $|x|\sqrt{5x}$

☐ $-|x|\sqrt{5x}$ ☐ $-5x\sqrt{x}$

14. Given that $x = 1$ is a zero of

$$p(x) = x^3 + x^2 - 10x + 8,$$

then one factor of $p(x)$ is

☐ $x^2 + 4x - 2$ ☐ $x^2 + x - 8$ ☐ $x^2 + 2x - 8$

☐ $x^2 - 2x - 2$ ☐ $x^2 - x - 4$

15. The inequality

$$|x - a| < \delta$$

is equivalent to

☐ $a - \delta < x < a + \delta$ ☐ $x < a - \delta \quad \text{or} \quad x > a + \delta$

☐ $x - a < \delta$ ☐ $x + a < \delta$

☐ None of these

16. $\dfrac{x^{3n}}{x^{n/3}}$ equals

☐ x^{9n} ☐ x^{3n} ☐ $x^{n/3}$

☐ $x^{8n/3}$ ☐ x^9

17. $\dfrac{x+1}{x-1} - \dfrac{x-1}{x+1}$ equals

☐ 1 ☐ -1 ☐ $\dfrac{2}{x^2-1}$

☐ 0 ☐ $\dfrac{4x}{x^2-1}$

18. $\sqrt[3]{\sqrt{2}}$ equals

☐ $\sqrt[5]{2}$ ☐ $\sqrt{2/3}$ ☐ $\sqrt[6]{2}$

☐ $2^{3/2}$ ☐ $2^{2/3}$

19. If $10 = 3e^t$, then $t =$

☐ $1-\sqrt{3}$ ☐ $\sqrt{3/10}$ ☐ 10^{3e}

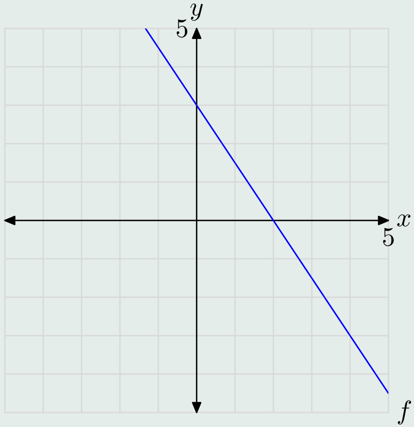
☐ $e^{10/3}$ ☐ $\ln(10/3)$

20. $(-3x^{-2}y^3)^{-2}$ equals

☐ $\dfrac{6x^4}{y^6}$ ☐ $\dfrac{6y}{x^4}$ ☐ $6xy^4$

☐ $\dfrac{x^4}{9y^6}$ ☐ $-\dfrac{6x^4}{y^6}$

21. Consider the following graph of f.



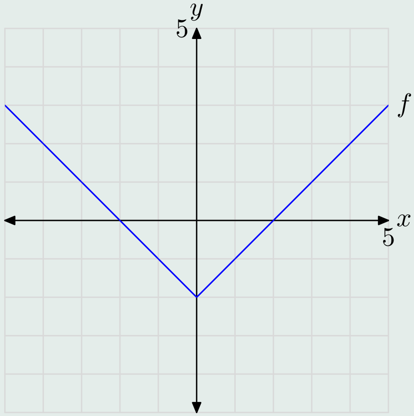
Which of the following is the equation of the line pictured above?

- ☐ $f(x) = \frac{2}{3}x - 3$
- ☐ $f(x) = \frac{2}{3}x + 3$
- ☐ $f(x) = \frac{3}{2}x + 3$
- ☐ $f(x) = -\frac{2}{3}x + 3$
- ☐ $f(x) = -\frac{3}{2}x + 3$

22. What is the equation of the line passing through the points $(2, 3)$ and $(5, 1)$?

- ☐ $y - 3 = -\frac{2}{3}(x - 5)$
- ☐ $y - 3 = \frac{3}{2}(x - 2)$
- ☐ $y - 3 = \frac{4}{7}(x - 2)$
- ☐ $y - 2 = -\frac{2}{3}(x - 3)$
- ☐ $y - 3 = -\frac{2}{3}(x - 2)$

23. What is the range of the function pictured below?



☐ $[0, +\infty)$ ☐ $(-\infty, +\infty)$ ☐ $[-2, +\infty)$

☐ $(-\infty, -2]$ ☐ $(-\infty, 0]$

24. If

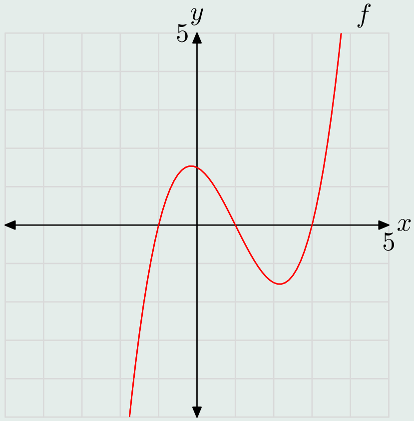
$$f(x) = \frac{x+2}{2x-3},$$

for what value of x is $f(x) = 2$?

☐ $3/8$ ☐ $8/3$ ☐ $2/5$

☐ $3/7$ ☐ $5/8$

25. One of the zeros of the polynomial



shown above is

- ☐ -4 ☐ -2 ☐ -1
- ☐ 0 ☐ 2

26. The inequality $4x^2 < 1$ is equivalent to

- ☐ $x < 1/2$ ☐ $x > 1/2$
- ☐ $x < -1/2 \quad \text{or} \quad x > 1/2$ ☐ $-1/2 < x < 1/2$
- ☐ None of these

27. If

$$8^{3x+1} = 4,$$

then $x =$

- ☐ $-1/3$
- ☐ $-5/3$
- ☐ $-2/9$
- ☐ $-2/3$
- ☐ $-1/9$

28. If

$$(x+3)^2 = -16,$$

then one solution is

- ☐ $3+4i$
- ☐ $3-4i$
- ☐ $-3+4i$
- ☐ $-3-8i$
- ☐ $-3+8i$

29. Given

$$f(x) = \frac{1}{x},$$

simplify

$$\frac{f(x)-f(2)}{x-2}.$$

- ☐ $\dfrac{2x}{x-2}$
- ☐ $\dfrac{x+2}{x-2}$
- ☐ $-\dfrac{1}{2x}$
- ☐ $\dfrac{-1}{2(x-2)}$
- ☐ $\dfrac{x-2}{2x}$

30. Solve

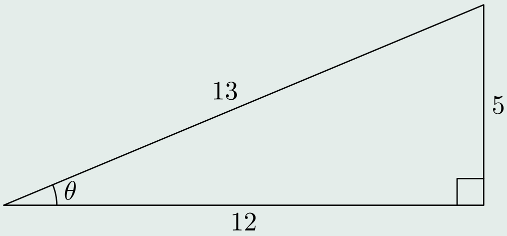
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

for x.

☐ $z - y$ ☐ yz ☐ $\dfrac{yz}{y+z}$

☐ $\dfrac{y+z}{yz}$ ☐ $\dfrac{yz}{y-z}$

31. Consider the following right triangle.



What is $\tan\theta$?

☐ $13/5$ ☐ $5/13$ ☐ $5/12$

☐ $13/12$ ☐ None of these

32. The perimeter of a rectangle is 20 meters. If the area of the rectangle is 100 square meters, which of the following equations best models the problem statement, given that L represents the length of the rectangle?

☐ $100 = L(L+10)$ ☐ $100 = L(L+20)$ ☐ $100 = 2L + 20$

☐ $100 = L(20-L)$ ☐ $100 = L(10-L)$

33. The sum of a number and its reciprocal is 3. One possible number satisfying this criteria is

☐ $2+\sqrt{2}$ ☐ $3-\sqrt{5}$ ☐ $\dfrac{3+\sqrt{5}}{2}$

☐ $\dfrac{2+\sqrt{3}}{5}$ ☐ $\dfrac{2-\sqrt{2}}{4}$

34. A rectangular garden is 60 feet by 80 feet. the outer edge of the garden is removed in order to install a walkway of uniform width about its perimeter. The area of the new garden is one-half its original area. Which of the following equations best models the problem statement, where x represents the uniform width of the walkway?

☐ $(80-x)(60-x)=2400$ ☐ $(80-2x)(60-2x)=4800$

☐ $4800-2x=2400$ ☐ $2(80-2x)+2(60-2x)=2400$

☐ $(80-2x)(60-2x)=2400$

35. If $\log_b 16 = 4$, then $\log_b 8 =$

☐ 0 ☐ 1 ☐ 2

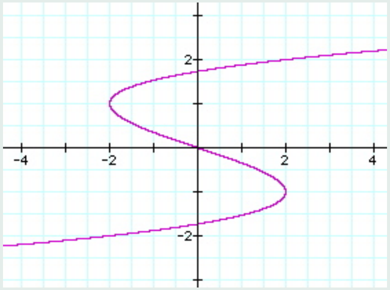
☐ 3 ☐ 4

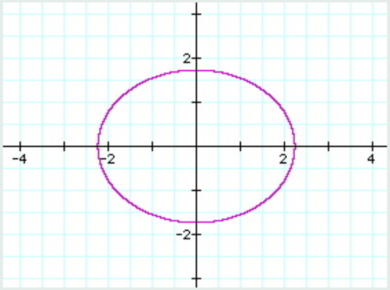
36. $x^{c-2}x^4 =$

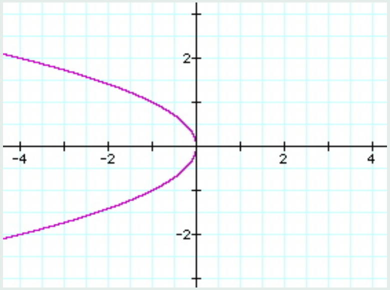
☐ x^{c-2^4} ☐ x^{c-6} ☐ x^{4c+2}

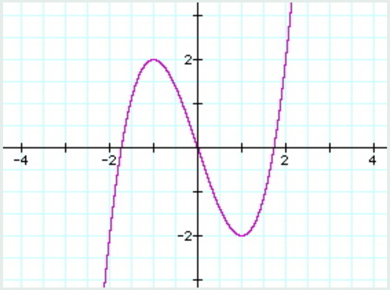
☐ x^{4c-8} ☐ x^{c+2}

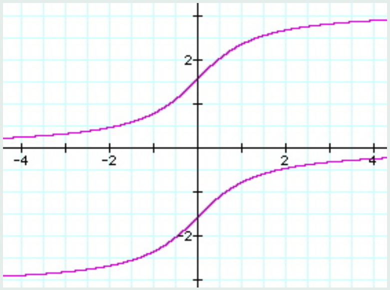
37. Which of the following could be a portion of the graph of a function $y = f(x)$?

☐ 

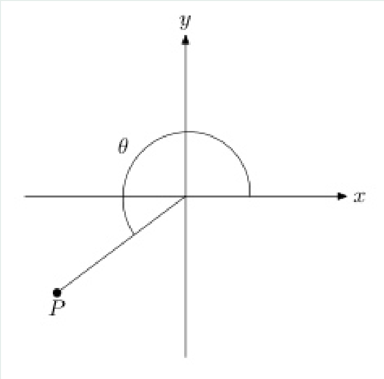
☐ 

☐ 

☐ 

☐ 

38. In the figure shown below, if P is the point $(-12, -9)$, then $\sec\theta =$



$\square\ -\dfrac{4}{5}$ $\square\ -\dfrac{5}{4}$ $\square\ -\dfrac{5}{3}$

$\square\ -\dfrac{3}{5}$ $\square\ -\dfrac{3}{4}$ $\square\ -\dfrac{4}{3}$

39. $\cos(\theta + \pi) =$

$\square\ -\cos\theta$ $\square\ -\sin\theta$ $\square\ \cos\theta$

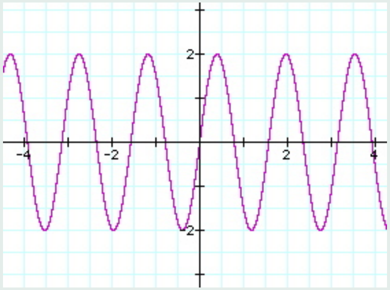
$\square\ \sin\theta$ $\square\ \cos 2\theta$

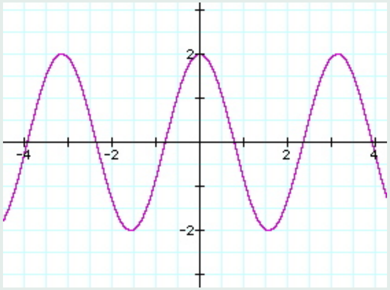
40. $\dfrac{z}{\sqrt[4]{2}} =$

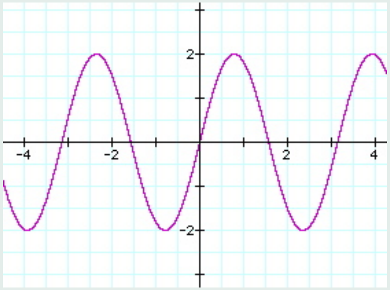
$\square\ \dfrac{z\sqrt[4]{2}}{2}$ $\square\ \dfrac{z\sqrt[4]{8}}{2}$ $\square\ \dfrac{z\sqrt[4]{8}}{8}$

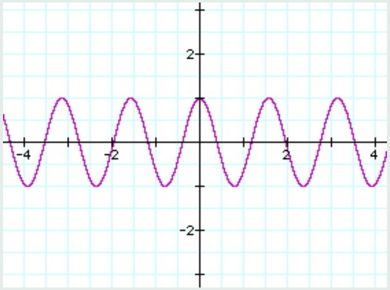
$\square\ \dfrac{z\sqrt{2}}{2}$ $\square\ \dfrac{\sqrt[4]{2z}}{2}$

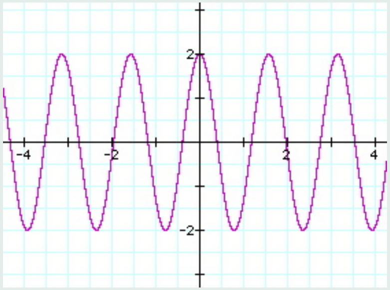
41. Which of the following could be a portion of the graph of $y = 2\cos 4x$?

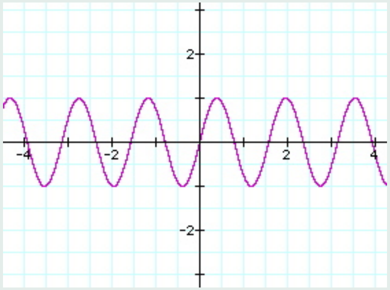
☐ 

☐ 

☐ 

☐ 

☐ 

☐ 

42. $\log_3 27 =$

☐ $\frac{1}{3}$ ☐ 2 ☐ 3
☐ 9 ☐ 81

43. If $c > 0$ and $x > 0$, then $\sqrt{9c^4x^2 + 27c^2x^4} =$

☐ $3c^2x + 3\sqrt{3}cx^2$ ☐ $9c^2x^2\sqrt{c^2 + 3x^2}$ ☐ $3cx\sqrt{c^2 + 3x^2}$
☐ $3c^2x\sqrt{1 + 27c^2x^4}$ ☐ $3cx^2\sqrt{c^4x^2 + 3}$

44. $\dfrac{3x + 6}{x^2 + 2x - 8} + \dfrac{x + 2}{2x + 8} =$

☐ $\dfrac{4x + 8}{x^2 + 4x}$ ☐ $\dfrac{x + 2}{2x - 4}$ ☐ $\dfrac{1}{2(x - 2)(x + 4)}$
☐ $\dfrac{3(x + 2)^2}{2(x - 2)(x + 4)}$ ☐ $\dfrac{(x + 2)(x + 8)}{2(x - 2)(x + 4)}$

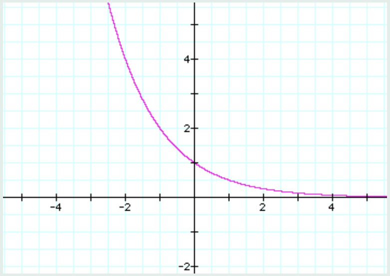
45. If the circumference of a circle is 10 centimeters, then the area, in square centimeters, is

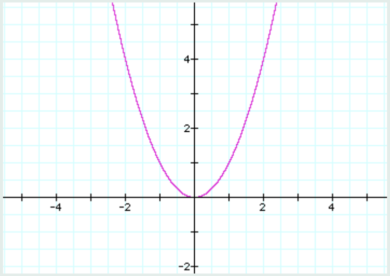
☐ 100π ☐ 25π ☐ $\frac{5}{\pi}$
☐ $\frac{25}{\pi}$ ☐ 20π

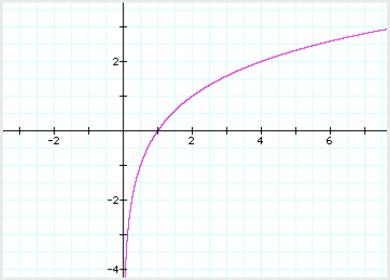
46. If $\ln z = 2 \ln x - 3 \ln y$, then $z =$

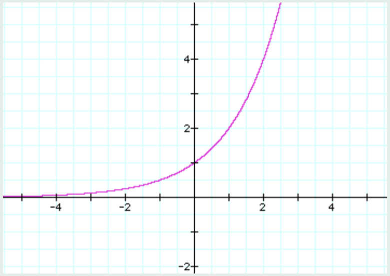
☐ $\dfrac{2x}{3y}$ ☐ $\dfrac{x^2}{y^3}$ ☐ $6xy$
☐ x^2y^3 ☐ $(x - y)^{2/3}$

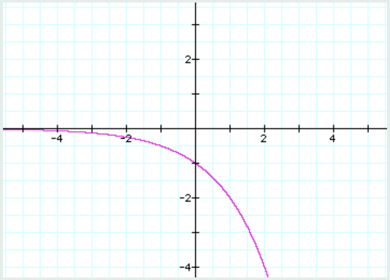
47. Which of the following could be a portion of the graph of $y = 2^x$?

☐ 

☐ 

☐ 

☐ 

☐ 

48. One of the roots of $x^2 + 3x + 1$ is

□ $\dfrac{-3+\sqrt{13}}{2}$ □ $\dfrac{3+\sqrt{5}}{2}$ □ 1

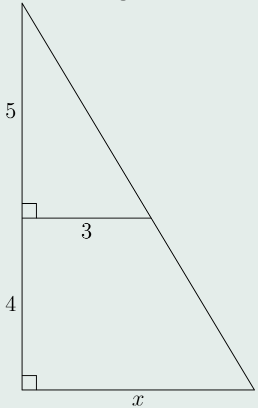
□ $\dfrac{3-\sqrt{13}}{2}$ □ -4 □ $\dfrac{-3-\sqrt{5}}{2}$

49. If $x \neq -3$ and $(x+3)(x-2)^2 - 4(x+3) = (x+3)R$, then $R =$

□ $(x-2)^2$ □ $x^2 - 4x$ □ $x^2 - 8x - 8$

□ $x^2 - 2x$ □ $-4(x-2)^2$

50. In the figure shown below, $x =$



□ $\dfrac{27}{4}$ □ 5 □ $\dfrac{27}{5}$

□ 6 □ $\dfrac{12}{5}$ □ $\dfrac{12}{4}$

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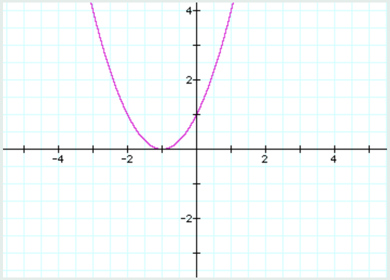
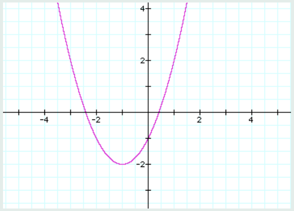
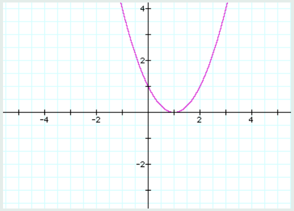
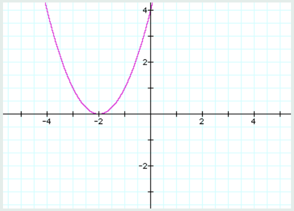
51. The graph of $y = f(x)$ is shown in the figure below.

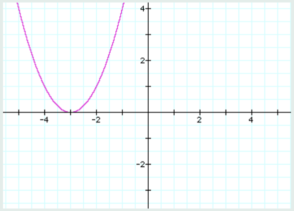


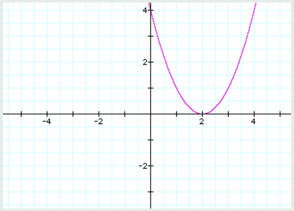
Which of the following is the graph of $y = f(x-2)$?

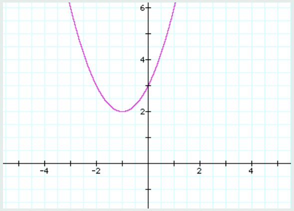
☐ 

☐ 

☐ 

☐ 

☐ 

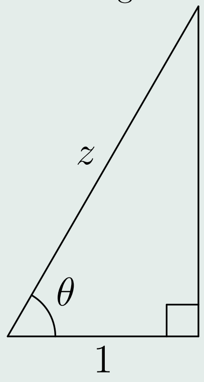
☐ 

52. If $f(x) = x^2 + 2x - 3$, then $f(b-2) =$

☐ $b^2 - 2b - 3$ ☐ $b^2 + 2b - 5$ ☐ $b^2 - 3$

☐ $b^2 - 2b - 5$ ☐ $b^2 + 2b - 3$

53. In the figure below, $\sin\theta =$



☐ $\dfrac{1}{z}$ ☐ $\dfrac{\sqrt{z^2+1}}{z}$ ☐ $\dfrac{1}{\sqrt{z^2-1}}$

☐ $\dfrac{\sqrt{z^2-1}}{z}$ ☐ $\dfrac{\sqrt{z^2+1}}{z}$ ☐ $\sqrt{z^2-1}$

54. If $\dfrac{\pi}{2} \leq x \leq \pi$ and $\sec^2 x = 2$, then $x =$

☐ $\dfrac{\pi}{4}$ ☐ $\dfrac{\pi}{2}$ ☐ $\dfrac{\pi}{3}$

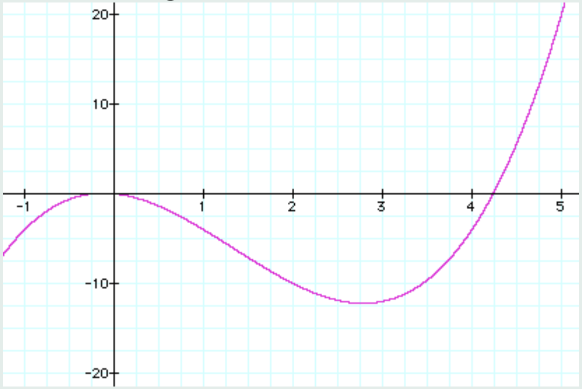
☐ $\dfrac{5\pi}{6}$ ☐ $\dfrac{2\pi}{3}$ ☐ $\dfrac{3\pi}{4}$

55. $\dfrac{\frac{1}{b} - \frac{1}{a}}{ab^2 - a^2b}$

☐ $-\dfrac{1}{ab}$ ☐ $-\dfrac{1}{a^2b^2}$ ☐ $\dfrac{1}{ab}$

☐ $\dfrac{1}{a^2b^2}$ ☐ $-(a-b)^2$

56. The graph of the function $y = f(x)$ is shown in the figure below. Which of the following is true?



☐ $f(0) < f(1)$ ☐ $f(2) > f(0)$ ☐ $f(2) < f(3)$

☐ $f(4) > f(3)$ ☐ $f(1) < f(3)$

57. The graphs of the two equations $-x + 3y = 2$ and $4x - 12y = -8$ are

- ☐ the same line
- ☐ two distinct parallel lines
- ☐ two perpendicular lines
- ☐ not straight lines
- ☐ two intersecting lines which are not perpendicular

58. The price of a pair of pants is on sale at 15% off the original price. If the sale price is \$22.91, what was the original price?

- ☐ 26.35
- ☐ 19.47
- ☐ 26.95
- ☐ 30.99
- ☐ 23.06

59. What is the degree measure of the angle whose radian measure is $\dfrac{4\pi}{9}$?

- ☐ 80
- ☐ 72
- ☐ 40
- ☐ 100
- ☐ 20

End Assessment

Score: Correct

Solutions to Assessment

Solution to Question 1: Substitute $x = -5$ to get

$$\begin{aligned} |x-3| - |4-3x| &= |-5-3| - |4-3(-5)| \\ &= |-8| - |4+5| \\ &= |-8| - |19| \\ &= 8 - 19 \\ &= -11. \end{aligned}$$

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Solution to Question 2: Raise each factor of the second factor to the third power.

$$(2x^2y^3)(-3x^3y^2)^3 = (2x^2y^3)((-3)^3(x^3)^3(y^2)^3)$$

Raising a power to another power requires that you repeat the base and multiply exponents $((a^m)^n = a^{mn})$.

$$= (2x^2y^3)(-27x^9y^6)$$

To multiply, repeat the common base and add exponents $(a^m \cdot a^n = a^{m+n})$.

$$-\,54x^{11}y^9$$

Click to return to assessment

Solution to Question 3: Jane can paint the room in 5 hours working alone. Therefore, each hours she finishes $1/5$ of the room. In 3 hours, she finishes $3/5$ of the room

Let L represent the time it takes Liz to paint the room working alone. Then, in one hour, Liz paints $1/L$ part of the room. In 3 hours, she finishes $3/L$ parts of the room.

Hence, working together,

$$\frac{3}{5} + \frac{3}{L} = 1,$$

where 1 represents the "whole" room.

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Solution to Question 4: First, substitute 3 in the function $g(x) = 3 - 2x$ to write

$$\begin{aligned} f(g(3)) &= f(3 - 2(3)) \\ &= f(3 - 6) \\ &= f(-3). \end{aligned}$$

Next, susbstitute -3 in the function $f(x) = 2x + 3$ to finish the calculation.

$$\begin{aligned} &= 2(-3) + 3 \\ &= -6 + 3 \\ &= -3 \end{aligned}$$

Click to return to assessment

Solution to Question 5: It's easier if you reduce before substituting. Factor and cancel.

$$\begin{aligned}\frac{a^2-2ab+b^2}{a^2-b^2} &= \frac{(a-b)^2}{(a+b)(a-b)}\\ &= \frac{a-b}{a+b}\end{aligned}$$

Let $a=2$ and $b=-7$.

$$\begin{aligned} &= \frac{2-(-7)}{2+(-7)}\\ &= \frac{2+7}{2+(-7)}\\ &= \frac{9}{-5}\\ &= -\frac{9}{5}\end{aligned}$$

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Solution to Question 6: Eliminate x.

$$x - y = a \tag{1}$$
$$x + 2y = b, \tag{2}$$

Subtract equation (2) from equation (1) to get

$$-3y = a - b$$

Dividing both sides by -3,

$$y = \frac{a-b}{-3}.$$

Negating both numerator and denominaotr,

$$y = \frac{b-a}{3}.$$

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Solution to Question 7: Cube both sides of the equation.

$$\sqrt[3]{3x-1} = -a$$
$$\left(\sqrt[3]{3x-1}\right)^3 = (-a)^3$$

Because $\left(\sqrt[3]{3x-1}\right)^3 = 3x-1$, we write

$$3x - 1 = -a^3.$$

Solve for x.

$$3x = 1 - a^3$$
$$x = \frac{1-a^3}{3}$$

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Solution to Question 8: Let B represent the number of blue marbles. Then $5B$ represents the number of red marbles. Note that the ratio of red to blue marles is then

$$\frac{5B}{B} = \frac{5}{1}.$$

Now, there are 90 marbles in all, so

$$5B + B = 90.$$

Solve for B.

$$\begin{aligned} 6B &= 90 \\ \frac{6B}{6} &= \frac{90}{6} \\ B &= 15 \end{aligned}$$

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Solution to Question 9: First, factor.

$$
\begin{aligned}
6x^2 - x - 15 &= 0 \\
(2x+3)(3x-15) &= 0
\end{aligned}
$$

At least one of these factors must equal zero.

$$2x + 3 = 0 \qquad \text{and} \qquad 3x - 5 = 0$$

Solving the first,

$$
\begin{aligned}
2x + 3 &= 0 \\
2x &= -3 \\
x &= -\frac{3}{2}.
\end{aligned}
$$

Solving the second,

$$
\begin{aligned}
3x - 5 &= 0 \\
3x &= 5 \\
x &= \frac{5}{3}.
\end{aligned}
$$

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Solution to Question 10: Factor numerators and denominators.

$$\frac{a^2+2ab+b^2}{a^2-b^2}\cdot\frac{a^2-ab}{5a+5b}=\frac{(a+b)^2}{(a+b)(a-b)}\cdot\frac{a(a-b)}{5(a+b)}$$

Cancel common factors.

$$=\frac{a}{5}$$

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Solution to Question 11: Start with

$$-x > 4$$

and multiply both sides by -1, remember to reverse the inequality (because you are multiplying by a negative number).

$$(-1)(-x) < (-1)(4)$$
$$x < -4$$

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Solution to Question 12: Start by raising each factor to the 1/2 power.

$$(4x^6y^{-4})^{1/2} = 4^{1/2}(x^6)^{1/2}(y^{-4})^{1/2}$$

Next, $4^{1/2} = \sqrt{4} = 2$, so

$$= 2(x^6)^{1/2}(y^{-4})^{1/2}$$

When raising a power to another power, repeat the base and multiply the exponents $((a^m)^n = a^{mn})$.

$$= 2x^{(6)(1/2)}y^{(-4)(1/2)}$$
$$= 2x^3y^{-2}$$

Finally, $y^{-2} = 1/y^2$, so

$$= 2x^3 \cdot \frac{1}{y^2}$$
$$= \frac{2x^3}{y^2}.$$

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Solution to Question 13: First,

$$\begin{aligned}\sqrt{25x^3} &= \sqrt{25x^2}\sqrt{x}\\ &= \sqrt{(5x)^2}\sqrt{x}.\end{aligned}$$

However, $\sqrt{(5x)^2}$ calls for the *nonnegative square root*, so $\sqrt{(5x)^2} = |5x|$ and we write

$$= |5x|\sqrt{x}.$$

But, $x < 0$, so $|5x| = -5x$, and we write

$$= -5x\sqrt{x}.$$

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Solution to Question 14: Because $x = 1$ is a zero of

$$p(x) = x^3 + x^2 - 10x + 8,$$

we know that $x - 1$ is a factor of $p(x)$. We can use either long division or synthetic division to find a second factor. We choose synthetic division.

$$\begin{array}{r|rrrr} \underline{1} & 1 & 1 & -10 & 8 \\ & & 1 & 2 & -8 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

Hence,

$$p(x) = (x - 1)(x^2 + 2x - 8).$$

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Solution to Question 15: The inequality $|y| < b$ is equivalent to $-b < y < b$. Using this structure, if

$$|x - a| < \delta,$$

then

$$-\delta < x - a < \delta.$$

Adding a to all three members of this last inequality,

$$a - \delta < x < a + \delta.$$

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Solution to Question 16: Repeat the base and subtract exponents.

$$\frac{x^{3n}}{x^{n/3}} = x^{3n-n/3}$$

Make equivalent fractions with a common denominator.

$$= x^{9n/3-n/3}$$

Subtract.

$$= x^{8n/3}$$

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Solution to Question 17: First, make equivalent fractions with a common denominator.

$$\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{(x+1)^2}{(x+1)(x-1)} - \frac{(x-1)^2}{(x+1)(x-1)}$$

Expand numerators.

$$= \frac{x^2+2x+1}{(x+1)(x-1)} - \frac{x^2-2x+1}{(x+1)(x-1)}$$

Subtract numerators.

$$\begin{aligned} &= \frac{(x^2+2x+1)-(x^2-2x+1)}{(x+1)(x-1)} \\ &= \frac{x^2+2x+1-x^2+2x-1}{(x+1)(x-1)} \\ &= \frac{4x}{(x+1)(x-1)} \end{aligned}$$

Expand the denominator.

$$= \frac{4x}{x^2-1}$$

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Solution to Question 18: Changing to fractional exponents,

$$\sqrt[3]{\sqrt{2}} = \left(2^{1/2}\right)^{1/3}.$$

Repeat the base an multiply exponents $((a^m)^n = a^{mn})$.

$$= 2^{(1/2)(1/2)}$$
$$= 2^{1/6}$$

Change back to radical notation.

$$= \sqrt[6]{2}.$$

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Solution to Question 19: Start with

$$10 = 3e^t$$

and divide both sides by 3.

$$e^t = \frac{10}{3}$$

Use the definition of the logarithm ($a^y = x$ if and only if $y = \log_a x$) to write

$$t = \log_e \frac{10}{3}.$$

Recall that we write ln in place of $\log_e$.

$$t = \ln \frac{10}{3}$$

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Solution to Question 20: Raise each factor to the power -2.

$$\left(-3x^{-2}y^{3}\right)^{-2} = (-3)^{-2}(x^{-2})^{-2}(y^{3})^{-2}$$

Recal that raising to a negative power is equivalent to taking the reciprocal (inverting). Hence, $(-3)^{-2} = 1/(-3)^{2} = 1/9$ and we write

$$= \frac{1}{9}(x^{-2})^{-2}(y^{3})^{-2}.$$

When raising a power to another power, repeat the base and mulitply exponents ($(a^{m})^{n} = a^{mn}$).

$$= \frac{1}{9}x^{4}y^{-6}$$

Finally, $y^{-6} = 1/y^{6}$, so

$$= \frac{1}{9}x^{4}\cdot\frac{1}{y^{6}}$$
$$= \frac{x^{4}}{9y^{6}}.$$

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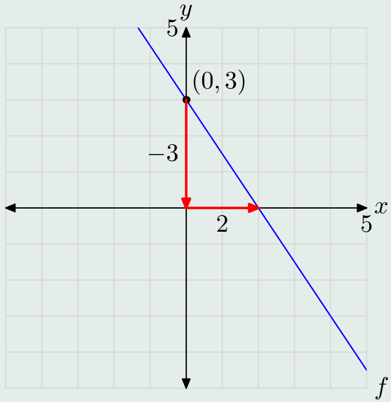
Solution to Question 21: First, note that the y-intercept of the line is $b = 3$. Secondly, use the graph to determine the slope of the line.



Thus, if m represents the slope of the line, then

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-3}{2} \\ 7 &= -\frac{3}{2}. \end{aligned}$$

Hence, the equation of the line is

$$\begin{aligned} f(x) &= mx + b \\ f(x) &= -\frac{3}{2}x + 3. \end{aligned}$$

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Solution to Question 22: First, find the slope of the line passing through $(2, 3)$ and $(5, 1)$.

$$m = \frac{1-3}{5-2} = \frac{-2}{3}.$$

Now, sub $m = -2/3$ and $(x_0, y_0) = (2, 3)$ into the point-slope form of the line.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 3 &= -\frac{2}{3}(x - 2) \end{aligned}$$

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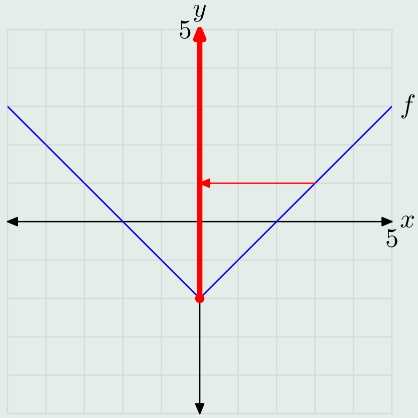
Solution to Question 23: To find the range of the given function, project all points on the graph onto the vertical axis to determine the y-value of each point (x, y) on the graph of f.



The shadow on the vertical axis is the set containing the y-values of each ordered pair (x, y) on the graph of f. This set is the range and is best described with

$$[-2, +\infty) = \{y : y \geq -2\}.$$

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Solution to Question 24: Given

$$f(x) = \frac{x+2}{2x-3},$$

then

$$f(x) = 2$$

becomes

$$\frac{x+2}{2x-3} = 2.$$

Multiply both sides by $2x-3$.

$$\begin{aligned}(2x-3)\left(\frac{x+2}{2x-3}\right) &= 2(2x-3)\\ x+2 &= 2(2x-3)\end{aligned}$$

Expand.

$$x+2 = 4x-6$$

Isolate x.

$$\begin{aligned}x-4x &= -2-6\\ -3x &= -8\end{aligned}$$

Divide both sides by -3.

$$x = \frac{8}{3}$$

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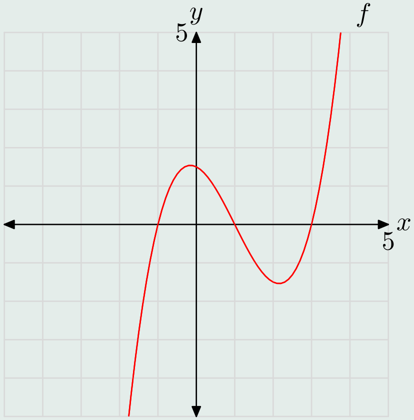
Solution to Question 25: Recall that x is a zero of a polynomial p if and only if $p(x) = 0$. The zeros are found by noting where the graph of p crosses the x-axis.



The graph of p crosses the x-axis at -1, 1, and 3. These are the zeros of the polynomial p.

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Solution to Question 26: Begin with

$$4x^2 < 1$$

and take the square root of both sides of the inequality.

$$\sqrt{4x^2} < \sqrt{1}$$

But $\sqrt{4x^2} = \sqrt{(2x)^2} = |2x|$, so this becomes

$$|2x| < 1.$$

Recall that $|y| < b$ if and only if $-b < y < b$. Use this structure to write

$$-1 < 2x < 1.$$

Divide through by 2.

$$-\frac{1}{2} < x < \frac{1}{2}$$

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Solution to Question 27: First, use a common base.

$$8^{3x+1} = 4$$
$$(2^3)^{3x+1} = 2^2$$

To raise a power to another power, repeat the base and multiply exponents $((a^m)^n = a^{mn})$.

$$2^{9x+3} = 2^2$$

EQuate exponents.

$$9x + 3 = 2$$

Solve for x.

$$9x = -1$$
$$x = -\frac{1}{9}$$

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Solution to Question 28: Recall that the solutions of $x^2 = b$ are $x = \pm\sqrt{b}$. Thus,

$$\begin{aligned}(x+3)^2 &= -16 \\ x+3 &= \pm\sqrt{-16}\end{aligned}$$

Now, $\sqrt{-16} = \sqrt{16}\,i = 4i$, so

$$x + 3 = \pm 4i$$

Subtract 3 from both sides of this last equation.

$$x = -3 \pm 4i$$

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Solution to Question 29: Start with

$$f(x) = \frac{1}{x},$$

and write

$$\frac{f(x) - f(2)}{x - 2} = \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

Multiply numerator and denominator by $2x$.

$$= \frac{2x\left(\frac{1}{x} - \frac{1}{2}\right)}{2x(x-2)}$$
$$= \frac{2 - x}{2x(x-2)}$$

Negate numerator and the fraction bar.

$$= -\frac{x - 2}{2x(x-2)}$$

Cancel.

$$= -\frac{1}{2x}$$

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Solution to Question 30: Multiply both sides by the common denominator.

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$
$$xyz\left(\frac{1}{x} + \frac{1}{y}\right) = xyz\left(\frac{1}{z}\right)$$
$$yz + xz = xy$$

Isolate terms with x on one side of the equation.

$$yz = xy - xz$$

Factor on the right.

$$yz = (y - z)x$$

Divide both sides by $y - z$.

$$x = \frac{yz}{y - z}$$

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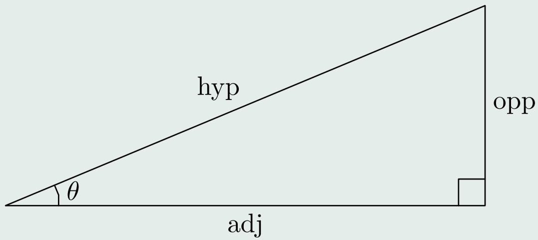
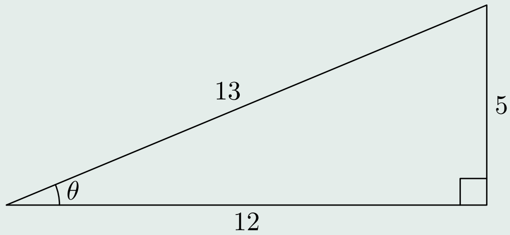
Solution to Question 31: When



then

$$\tan\theta = \frac{\text{opp}}{\text{hyp}}.$$

In this case,



we see that

$$\tan\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{12}.$$

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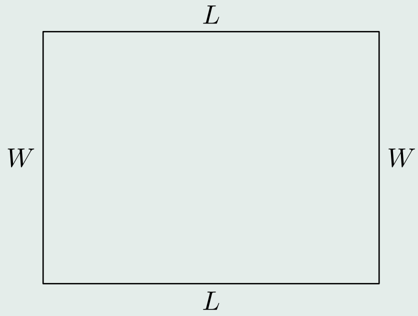
Solution to Question 32: The perimeter of the rectangle is found by adding the four sides of the rectangle.



If the perimeter of the rectangle is 20 meters, then

$$\begin{aligned} W + L + W + L &= 20 \\ 2W + 2L &= 20. \end{aligned}$$

Dividing by 2,

$$W + L = 10,$$

or equivalently,

$$W = 10 - L.$$

The area is given by

$$A = LW$$

If the are is $100\,\mathrm{m}^2$, then

$$100 = LW.$$

Subbing $W = 10 - L$,

$$100 = L(10 - L).$$

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Solution to Question 33: Let x represent a number, then its reciprocal is $1/x$. Now the sum of these two numbers is 3, so we write

$$x + \frac{1}{x} = 3.$$

Multiply both sides by x.

$$\begin{aligned} x\left(x + \frac{1}{x}\right) &= 3x \\ x^2 + 1 &= 3x \end{aligned}$$

Make one side zero.

$$x^2 - 3x + 1 = 0$$

The quadratic formula provides the solution.

$$\begin{aligned} x &= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \\ x &= \frac{3 \pm \sqrt{9-4}}{2} \\ x &= \frac{3 \pm \sqrt{5}}{2} \end{aligned}$$

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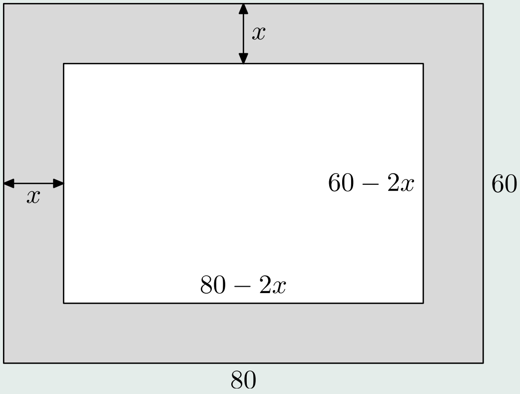
Solution to Question 34: Let x represent the uniform width of the walkway. Then a picture is worth a thousand words.



The area of the original garden is

$$80\,\text{ft} \times 60\,\text{ft} = 4800\,\text{ft}^2.$$

The new garden has area $2400\,\text{ft}^2$ (half the original area). Thus,

$$(80 - 2x)(60 - 2x) = 2400.$$

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Solution to Question 35: $\log_b 16 = 4 \iff b^4 = 16 \iff b = 2.$
Then $\log_b 8 = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3.$

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Solution to Question 36: $x^{c-2}x^4 = x^{c-2+4} = x^{c+2}$

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Solution to Question 37: The graph of a function must satisfy the *vertical line test*: every vertical line intersects the graph at most once. The only choice which satisfies the vertical line test is choice #4.

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Solution to Question 38: $\cos\theta = \dfrac{-12}{\sqrt{(-12)^2 + (-9)^2}} = \dfrac{-12}{\sqrt{225}} = -\dfrac{12}{15} = -\dfrac{4}{5}$, so $\sec\theta = -\dfrac{5}{4}$.

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Solution to Question 39:

$$\cos(\theta + \pi) = \cos\theta\cos\pi - \sin\theta\sin\pi = (\cos\theta)(-1) - (\sin\theta)(0) = -\cos\theta$$

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Solution to Question 40: $\dfrac{z}{\sqrt[4]{2}} = \dfrac{z}{2^{1/4}} = \dfrac{z \cdot 2^{3/4}}{2^{1/4}2^{3/4}} = \dfrac{z\sqrt[4]{8}}{2}$

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Solution to Question 41: $f(x) = 2\cos 4x$ has amplitude 2, period $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$ and $f(0) = 2$. The only graph with these properties is choice #5.

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Solution to Question 42: $\log_3 27 = \log_3 3^3 = 3\log_3 3 = 3$

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Solution to Question 43: $\sqrt{9c^4x^2 + 27c^2x^4} = \sqrt{9c^2x^2(c^2 + 3x^2)} = 3cx\sqrt{c^2 + 3x^2}$

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Solution to Question 44:

$$
\begin{aligned}
&\frac{3x+6}{x^2+2x-8}+\frac{x+2}{2x+8}\\
=&\frac{3(x+2)}{(x+4)(x-2)}+\frac{x+2}{2(x+4)}\\
=&\frac{6(x+2)}{2(x+4)(x-2)}+\frac{(x+2)(x-2)}{2(x+4)(x-2)}\\
=&\frac{6(x+2)+(x+2)(x-2)}{2(x+4)(x-2)}\\
=&\frac{x^2+6x+8}{2(x+4)(x-2)}\\
=&\frac{(x+4)(x+2)}{2(x+4)(x-2)}\\
=&\frac{(x+2)}{2(x-2)}\\
=&\frac{x+2}{2x-4}
\end{aligned}
$$

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Solution to Question 45: $2\pi r = 10 \implies r = \dfrac{5}{\pi} \implies \text{area} = \pi r^2 = \dfrac{25}{\pi}$

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Solution to Question 46:

$$
\begin{aligned}
&\ln z = 2\ln x - 3\ln y \\
\Longrightarrow\ &\ln z = \ln x^2 - \ln y^3 \\
\Longrightarrow\ &\ln z = \ln\left(\frac{x^2}{y^3}\right) \\
\Longrightarrow\ &z = \frac{x^2}{y^3}
\end{aligned}
$$

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Solution to Question 47: The graph of $y = a^x$ with $a > 1$ is increasing, always positive, and passes through the point $(0, 1)$. The only graph with these properties is choice #4.

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Solution to Question 48: Applying the quadratic formula yields $\dfrac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$, so the last choice is the correct answer.

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Solution to Question 49: $(x+3)(x-2)^2 - 4(x+3) = (x+3)((x-2)^2 - 4) = (x+3)(x^2 - 4x)$, so $R = x^2 - 4x$.

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Solution to Question 50: By properties of similar triangles, $\dfrac{x}{4+5} = \dfrac{3}{5} \implies x = \dfrac{27}{5}$.

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Solution to Question 51: The graph of $f(x-2)$ is the same as the graph of $f(x)$ shifted to the right 2 units. Thus, the correct choice is choice #2.

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Solution to Question 52:

$$f(x) = x^2 + 2x - 3$$
$$\Longrightarrow f(b-2) = (b-2)^2 + 2(b-2) - 3 = b^2 - 4b + 4 + 2b - 4 - 3 = b^2 - 2b - 3$$

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Solution to Question 53: The other leg of the triangle has length $\sqrt{z^2 - 1}$, so $\sin\theta = \dfrac{\sqrt{z^2-1}}{z}$.

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Solution to Question 54: $\sec^2 x = 2 \implies \cos^2 x = \dfrac{1}{2} \implies \cos x = \pm\dfrac{1}{\sqrt{2}}$. Since $\dfrac{\pi}{2} \leq x \leq \pi$, it follows that $\cos x = -\dfrac{1}{\sqrt{2}}$, and therefore $x = \dfrac{3\pi}{4}$.

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Solution to Question 55:

$$\begin{aligned}
\frac{\frac{1}{b}-\frac{1}{a}}{ab^2-a^2b} &= \frac{\frac{a-b}{ab}}{ab(b-a)} \\
&= \frac{a-b}{a^2b^2(b-a)} \\
&= -\frac{b-a}{a^2b^2(b-a)} \\
&= -\frac{1}{a^2b^2}
\end{aligned}$$

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Solution to Question 56: $f(0) = 0$, $f(1) \approx -4$, $f(2) \approx -10$, $f(3) \approx -12$, and $f(4) \approx -4$. Therefore, $f(4) > f(3)$ and all of the other inequalities are false.

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Solution to Question 57: $-x + 3y = 2 \implies 3y = x + 2 \implies y = \dfrac{x}{3} + \dfrac{2}{3}$ and $4x - 12y = -8 \implies 12y = 4x + 8 \implies y = \dfrac{x}{3} + \dfrac{2}{3}$. Therefore, the two lines are actually the same.

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Solution to Question 58: $22.91 = x - .15x \implies 22.91 = .85x \implies x = \frac{22.91}{.85} \implies x \approx 26.95$

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Solution to Question 59: $\frac{4\pi}{9} \cdot \frac{180}{\pi} = 80$

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